Class X Session 2024-25
Subject - Mathematics (Basic)
Sample Question Paper - 11

Time: 3 hours. **Total Marks: 80**

General Instructions:

1. This Question Paper has 5 Sections A - E.
2. Section A has 18 multiple choice questions and 2 Assertion-Reason based questions carrying 1 mark each.
3. Section B has 5 questions carrying 02 marks each.
4. Section C has 6 questions carrying 03 marks each.
5. Section D has 4 questions carrying 05 marks each.
6. Section E has 3 case study-based questions carrying 4 marks each with subparts of 1, 1, and 2 marks each, respectively.
7. All Questions are compulsory. However, an internal choice in 2 Question of Section B, 2 Questions of Section C and 2 Questions of Section D has been provided. An internal choice has been provided in all the 2 marks questions of Section E.
8. Draw neat figures wherever required. Take $\pi = 22/7$ wherever required if not stated.

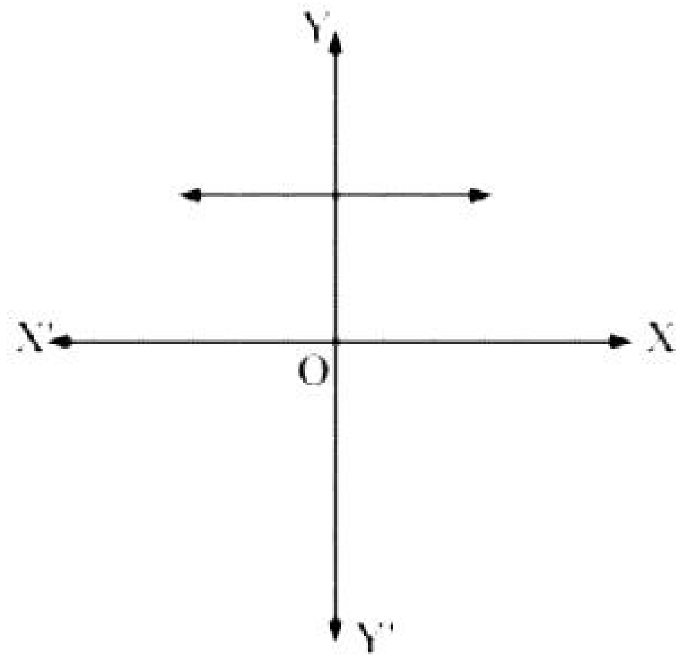
Section A
Section A consists of 20 questions of 1 mark each.

Choose the correct answers to the questions from the given options. [20]

1. Find the HCF of 196 and 38220.
(a) 196
(b) 195
(c) 194
(d) 193

2. Which of the following is an irrational number?
(a) $\sqrt{4}$
(b) $\sqrt{6}$
(c) $\sqrt{9}$
(d) $\sqrt{49}$

3. Find the number of zeroes of p(x), in the following case.



(a) 1
(b) 2
(c) 3
(d) 0

4. Find the product of zeroes of the polynomial $3x^2 - x - 4 = 0$.
(a) 1/3
(b) 4/3
(c) −1/3
(d) −4/3

5. The quadratic polynomial with zeroes 3 and −2 is ...
(a) $x^2 + x + 6$
(b) $x^2 - x - 6$
(c) $x^2 - x + 6$
(d) $x^2 + x - 6$

6. Find the distance between the points (−5, 7) and (−1, 3).
(a) $\sqrt{2}$
(b) $2\sqrt{2}$
(c) $4\sqrt{2}$
(d) $2\sqrt{3}$

7. A line intersecting a circle at two points is called a ________.
(a) tangent
(b) radius
(c) secant
(d) diameter

8. A circle can have ________ parallel tangents at the most.
(a) 2
(b) 3
(c) 4
(d) 5

9. The common point of a tangent to a circle and the circle is called___________.

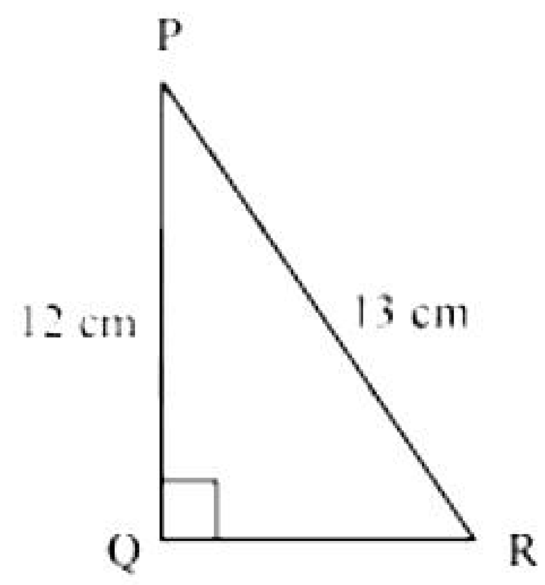
(a) point of contact
(b) intersection point
(c) touching point
(d) common point

10. In ΔABC right angled at B, AB = 24 cm, BC = 7 m. Determine cos A.

(a) 7/24
(b) 7/25
(c) 24/25
(d) 24/7

11. In the given figure, find cot P.

(a) 13/12
(b) 12/5
(c) 5/13
(d) 5/12



12. Given 15 cot A = 8. Find tan A.

(a) 17/15
(b) 8/15
(c) 15/17
(d) 15/8

13. Find the side length of a cube having volume 64 cm^3.

(a) 4 cm
(b) 6 cm
(c) 8 cm
(d) 12 cm

14. A chord of a circle of radius 10 cm subtends a right angle at the centre. Find the area of the corresponding minor segment.

(a) 76.5 cm^2
(b) 75.5 cm^2
(c) 77.5 cm^2
(d) 78.5 cm^2

15. Probability of an event that cannot happen is _________.

(a) 0
(b) 1/2
(c) 1/4
(d) 1

16. The probability of any event is always less than or equal to_____

(a) 0

(b) 1/2

(c) 1/4

(d) 1

17. In a circle of radius 21 cm, an arc subtends an angle of 60° at the centre. Find the length of the arc.

(a) 22 cm

(b) 11 cm

(c) 44 cm

(d) 21 cm

18. If P(E) = 0.05, what is the probability of 'not E'?

(a) 0.5

(b) 0.75

(c) 0.9

(d) 0.95

DIRECTION: In the question number 19 and 20, a statement of **Assertion (A)** is followed by a statement of **Reason (R)**. Choose the correct option.

19. Statement A (Assertion): The points (1, 5), (2, 3) and (–2, –11) are collinear.
Statement R (Reason): Three points are collinear if they lie on a same line i.e., one point lies in between any other two points.

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A)

(b) Both assertion (A) and reason (R) are true and reason (R) is not the correct explanation of assertion (A)

(c) Assertion (A) is true but reason (R) is false.

(d) Assertion (A) is false but reason (R) is true.

20. Statement A (Assertion): If HCF (115, 25) = 5, then LCM (115, 25) = 525.
Statement R (Reason): Product of HCF and LCM of two numbers is equal to the product of the numbers.

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A)

(b) Both assertion (A) and reason (R) are true and reason (R) is not the correct explanation of assertion (A)

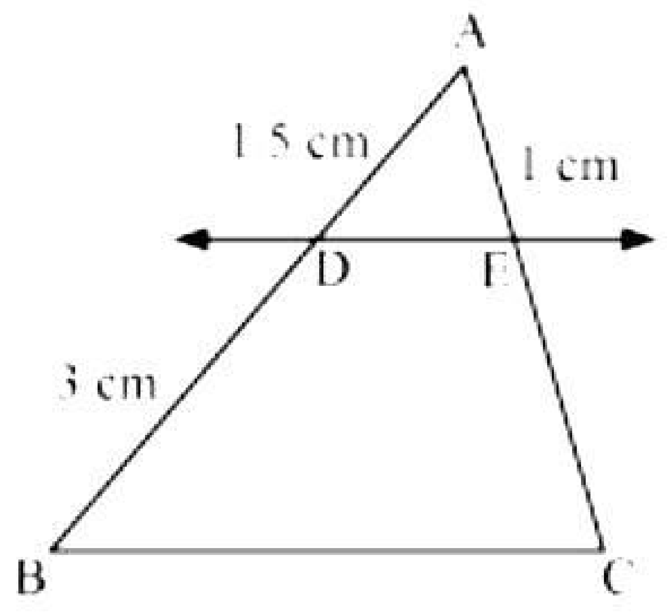
(c) Assertion (A) is true but reason (R) is false.

(d) Assertion (A) is false but reason (R) is true.

Section B

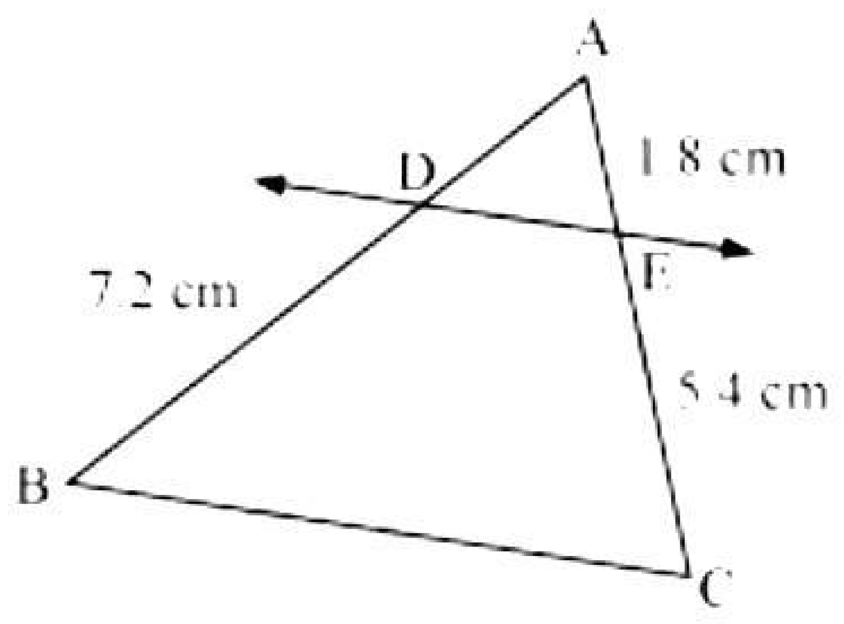
21. Find the zeroes of the quadratic polynomial $6x^2 - 3 - 7x$ and verify the relationship between the zeroes and the coefficients. [2]

22. In figure below, DE || BC. Find EC. [2]

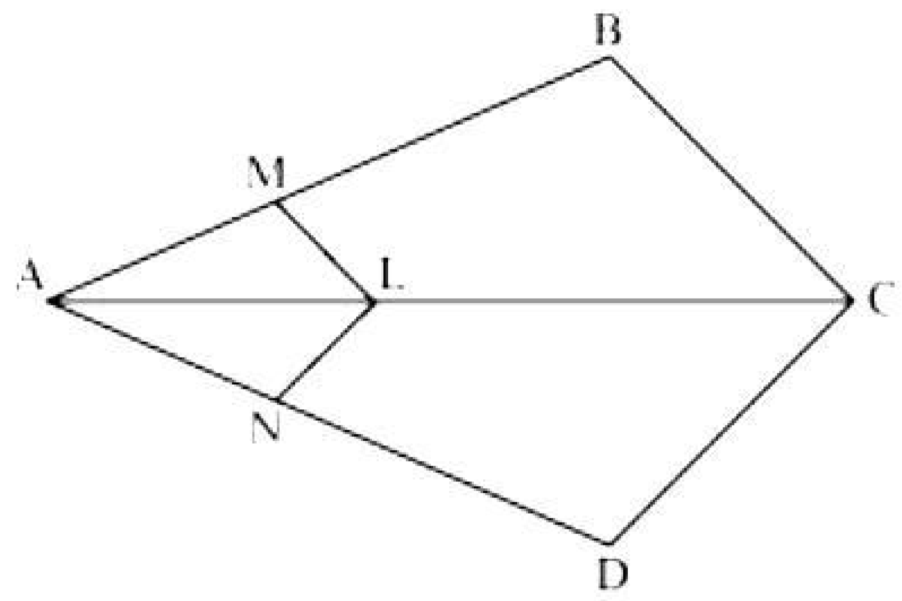


OR

In figure below, DE || BC. Find AD.



23. In figure, if LM || CB and LN || CD, prove that $\frac{AM}{AB} = \frac{AN}{AD}$. [2]



24. If $\angle A$ and $\angle B$ are acute angles in right-angled $\triangle ABC$, such that $\cos A = \cos B$, then show that $\angle A = \angle B$. [2]

25. A vessel is in the form of a hollow hemisphere mounted by a hollow cylinder. The diameter of the hemisphere is 14 cm and the total height of the vessel is 13 cm. Find the inner surface area of the vessel. [2]

OR

A hemispherical depression is cut out from one face of a cubical wooden block such that the diameter l of the hemisphere is equal to the edge of the cube. Determine the surface area of the remaining solid.

Section C

Section C consists of 6 questions of 3 marks each.

26. Show that any number of the form 4^n, $n \in N$ can never end with the digit 0. [3]

27. Five years hence, the age of Jacob will be three times that of his son. Five years ago, Jacob's age was seven times that of his son. What is their present age? [3]

28. The sum of the digits of a two-digit number is 9. Also, nine times this number is twice the number obtained by reversing the order of the digits. Find the number. [3]

OR

Meena went to a bank to withdraw Rs. 2000. She asked the cashier to give her Rs. 50 and Rs. 100 notes only. Meena got 25 notes in all. Find how many notes of Rs. 50 and Rs. 100 she received.

29. The diagonals of a quadrilateral ABCD intersect each other at point O such that $\frac{AO}{BO} = \frac{CO}{DO}$. Show that ABCD is a trapezium. [3]

30. From a point on the ground, the angles of elevation of the bottom and the top of a transmission tower fixed at the top of a 20 m high building are 45° and 60° respectively. Find the height of the tower. [3]

OR

A statue, 1.6 m tall, stands on the top of pedestal. From a point on the ground, the angle of elevation of the top of statue is 60° and from the same point the angle of elevation of the top of the pedestal is 45°. Find the height of the pedestal.

31. Consider the following distribution of daily wages of 50 workers in a factory. [3]

Daily wages (in Rs.)	100–120	120–140	140–160	160–180	180–200
Number of workers	12	14	8	6	10

Find the mean daily wages of the workers of the factory by using an appropriate method.

Section D

Section D consists of 4 questions of 5 marks each.

32. If the 3rd and the 9th terms of an A.P. are 4 and –8 respectively, which term of this A.P. is zero? [5]

OR

Two A.P.s have the same common difference. The difference between their 100th terms is 100, what is the difference between their 1000th terms?

33. Prove that the opposite sides of a quadrilateral circumscribing a circle subtend supplementary angles at the centre of the circle. [5]

34. Rachel, an engineering student, was asked to make a model shaped like a cylinder with two cones attached at its two ends by using a thin aluminium sheet. The diameter of the model is 3 cm and its length is 12 cm. if each cone has a height of 2 cm, find the volume of air contained in the model that Rachel made. [5]

OR

A vessel is in the form of an inverted cone. Its height is 8 cm and the radius of its top, which is open, is 5 cm. it is filled with water up to the brim. When lead shots, each of which is a sphere of radius 0.5 cm are dropped into the vessel, one-fourth of the water flows out. Find the number of lead shots dropped in the vessel.

35. 100 surnames were randomly picked up from a local telephone directory and the frequency distribution of the number of letters in the English alphabets in the surnames was obtained as follows: [5]

Number of letters	1 – 4	4 – 7	7 – 10	10 – 13	13 – 16	16 – 19
Number of surnames	6	30	40	6	4	4

Determine the median number of letters in the surnames. Find the mean number of letters in the surnames? Also, find the modal size of the surnames.

Section E

Case study-based questions are compulsory.

36. It is festival time; all smartphone companies have come up with no cost EMI plans to sell their flagship models. Now, Aarushi always wanted to buy a flagship smartphone, so she decided to take advantage of this offer. She buys a smart phone on EMI of Rs. 1000 per month. She pays Rs. 1000 for the first month and decides to make the subsequent payment in such a manner that the current month's payment will be always Rs. 100 more than the previous month. Now using the information given, answer the following questions.

i. Find the amount paid by her in 30th month. [1]

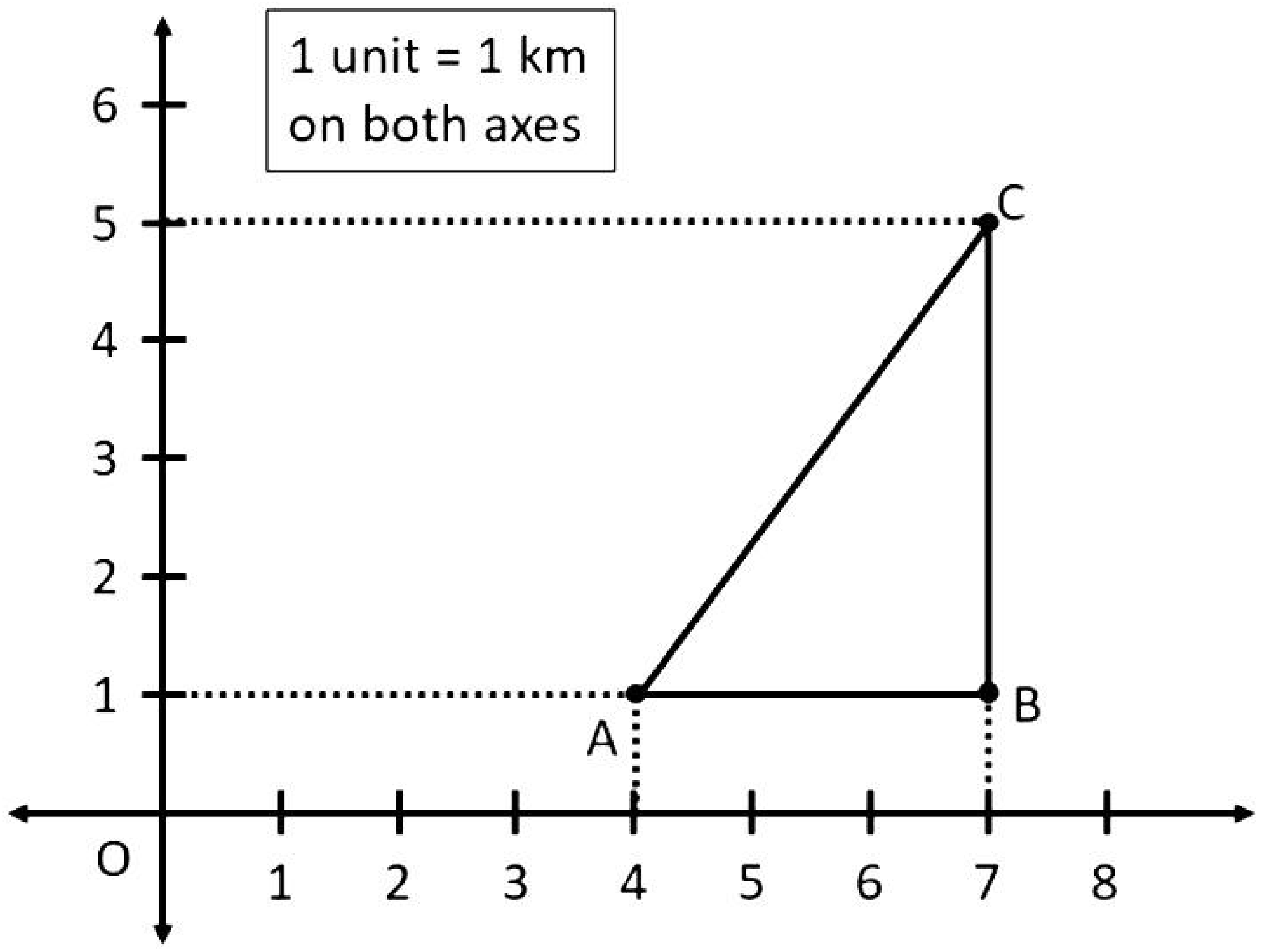
ii. For a particular month, Aarushi pays Rs. 4900 as instalment, find which month is this. [1]

iii. Find the ratio of the payments made in 19th month to the 28th month. [2]

OR

Find the total amount paid by Aarushi in 30 months. [2]

37. The location of homes of three friends Ajay, Bipin and Chandu are shown by the points A, B and C respectively. Now using the given information, answer the following questions.



i. Find the distance between Ajay's house and Bipin's house. [1]

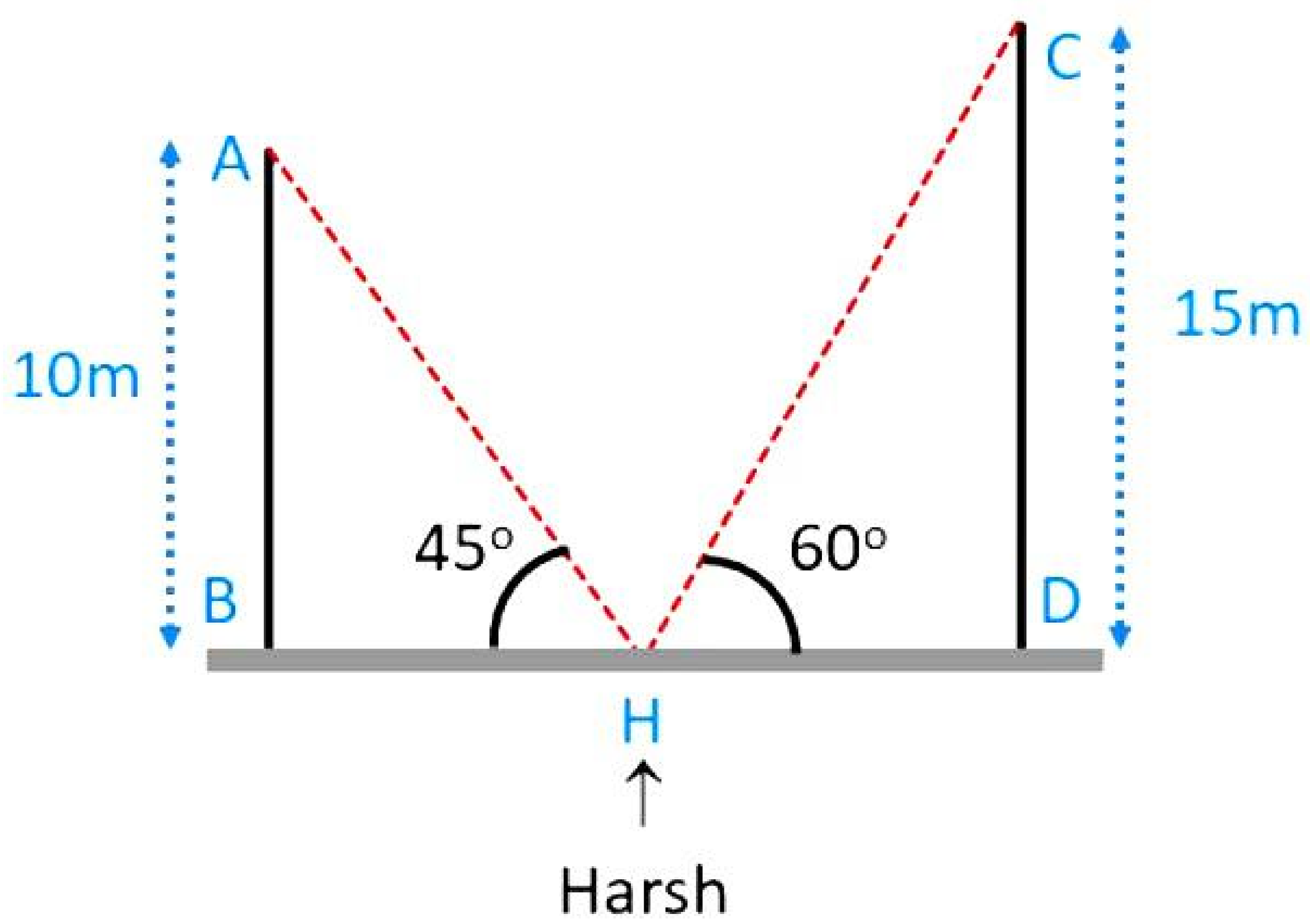
ii. Find the distance between Chandu's house and Bipin's house. [1]

iii. Find the distance between Chandu's house and Ajay's house. [2]

OR

Find the difference between the longest and the shortest route from Ajay's house to Chandu's house.

38. Harsh is standing between two buildings having height 10 m (AB) and 15 m (CD). Now the angle of elevation from the point (H) where harsh is standing, to the top of 10 m building is 45°, whereas the angle of elevation from the same point to the top of 15 m building is 60°. Using the given data, answer the following questions.



i. Find the distance between harsh and building AB. [1]

ii. Find the distance between harsh and building CD. [1]

iii. Find the length of a rope joining points A and H. [2]

OR

Find the length of a rope joining points C and H.

Solution

Section A

1. Correct option: (a)
Explanation:
$196 = 2 \times 2 \times 7 \times 7$
$38220 = 2 \times 2 \times 3 \times 5 \times 7 \times 7 \times 13$
$HCF(196, 38220) = 2^2 \times 7^2 = 196$

2. Correct option: (b)
Explanation:
$\sqrt{4} = 2$
$\sqrt{9} = 3$
$\sqrt{49} = 7$
But, $\sqrt{6}$ cannot be expressed in a rational form.
Hence, $\sqrt{6}$ is an irrational number.

3. Correct option: (d)
Explanation:
The graph of p(x) does not cut the x-axis at any point. So, the number of zeroes is 0.

4. Correct Option: (d)
Explanation:
$$\text{Product of zeroes } = \frac{\text{Constant term}}{\text{Coefficient of } x^2} = \frac{-4}{3}$$

5. Correct Option: (b)
Explanation:
Let the polynomial be $ax^2 + bx + c$.
$$\alpha + \beta = 1 = \frac{1}{1} = \frac{-b}{a} \text{ and } \alpha \times \beta = -6 = \frac{-6}{1} = \frac{c}{a}$$
If $a = k$, then $b = -k$, $c = -6k$
Therefore, the quadratic polynomial is $k(x^2 - x - 6)$, where k is a real number.

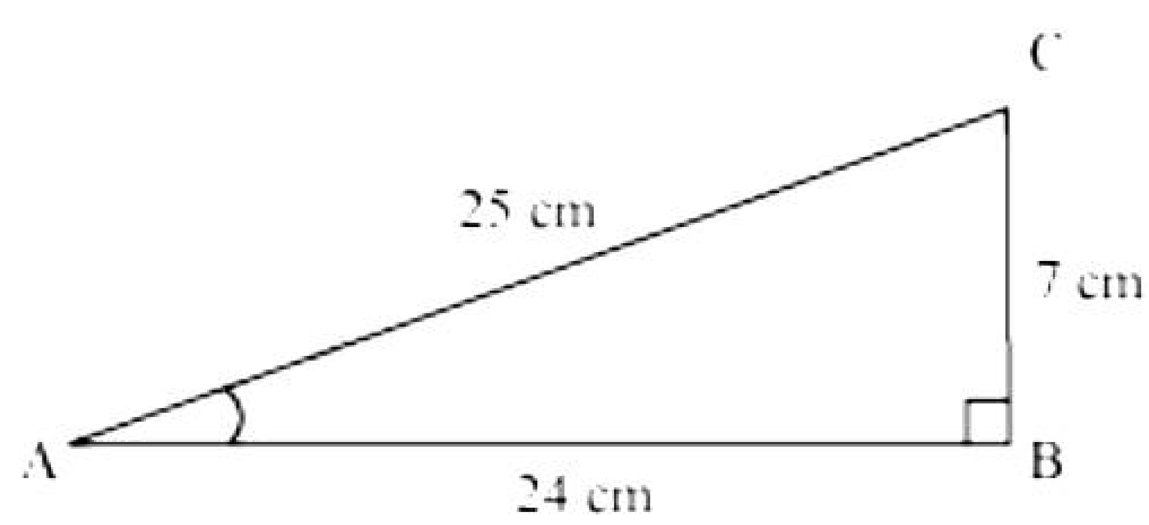
6. Correct Option: (c)
Explanation:
Distance between the points) (–5, 7) and (–1, 3) is given by
$$D = \sqrt{(-5-(-1))^2 + (7-3)^2} = \sqrt{(-4)^2 + (4)^2} = \sqrt{16+16} = \sqrt{32} = 4\sqrt{2}$$

7. Correct option: (c)
Explanation:
A line intersecting a circle at two points is called a **secant.**

8. Correct option: (a)
Explanation:
A circle can have **2** parallel tangents at the most.

9. Correct Option: (a)
Explanation:
The common point of a tangent to a circle and the circle is called **point of contact**.

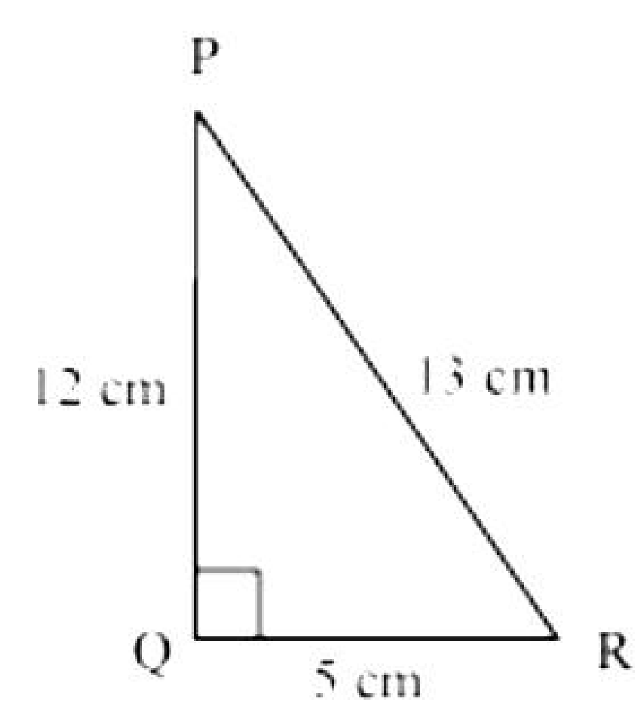
10. Correct Option: (c)
Explanation:
In ΔABC, by applying Pythagoras theorem,



$AC^2 = AB^2 + BC^2$
$= (24)^2 + (7)^2$
$= 576 + 49$
$= 625$
$\therefore AC = \sqrt{625} = 25$ cm

$$\cos A = \frac{\text{side adjacent to } \angle A}{\text{hypotenuse}} = \frac{AB}{AC} = \frac{24}{25}$$

11. Correct option: (b)
Explanation:
In ΔPQR, by applying Pythagoras theorem,



$PR^2 = PQ^2 + QR^2$
$(13)^2 = (12)^2 + QR^2$
$= 144 + QR^2$
$25 = QR^2$
$\therefore QR = 5$ cm

$$\cot P = \frac{\text{Side adjacent to } \angle P}{\text{Side opposite to } \angle P} = \frac{PQ}{QR} = \frac{12}{5}$$

12. Correct Option: (d)
Explanation:
$15 \cot A = 8$

$$\cot A = \frac{8}{15}$$

$$\tan A = \frac{1}{\cot A}$$

$$\therefore \tan A = \frac{15}{8}$$

13. Correct Option: (a)
Explanation:
Volume of cube = 64 cm^3
∴ $(side)^3 = 64\ cm^3$
∴ side = 4 cm

14. Correct option: (d)
Explanation:
Area of minor sector = $\frac{90°}{360°} \times \pi r^2$
$= \frac{1}{4} \times \frac{22}{7} \times 10 \times 10$
$= \frac{1100}{14} = 78.5\ cm^2$

15. Correct option: (a)
Explanation:
Probability of an event that cannot happen is **0**.

16. Correct Option: (d)
Explanation:
The probability of any event is always less than or equal to **1**.

17. Correct option: (a)
Explanation:
Radius (r) of circle = 21 cm
Angle subtended by given arc = 60°
Length of an arc of a sector of angle θ = $\frac{\theta}{360°} \times 2\pi r$
$= \frac{60°}{360°} \times 2 \times \frac{22}{7} \times 21$
$= \frac{1}{6} \times 2 \times 22 \times 3$
= 22 cm

18. Correct Option: (d)
Explanation:
$P(\bar{E}) = 1 - P(E) = 1 - 0.05 = 0.95$

19. Correct Option: (d)
Explanation:
Three points are collinear if they lie on a same line i.e., one point lies in between any other two points.
Hence, the reason is true.

Let A = (1, 5), B = (2, 3) and C = (−2, −11)

Therefore $AB = \sqrt{(1-2)^2 + (5-3)^2} = \sqrt{5}$

$BC = \sqrt{(2-(-2))^2 + (3-(-11))^2} = \sqrt{4^2 + 14^2} = \sqrt{16+196} = \sqrt{212}$

$CA = \sqrt{(1-(-2))^2 + (5-(-11))^2} = \sqrt{3^2 + 16^2} = \sqrt{9+256} = \sqrt{265}$

Here, the sum of the two distances is not equal to the third distance.
Therefore, points (1, 5), (2, 3) and (−2, −11) are not collinear.
Hence, the assertion is false.

20. Correct Option: (d)
Explanation:
According to the question,
HCF (115, 25) = 5
We know that a product of HCF and LCM of two numbers equals the product of the numbers.
So, the reason is true.
⇒ HCF × LCM = Product of numbers

$\Rightarrow \text{LCM} = \frac{115 \times 25}{5} \Rightarrow \text{LCM} = 575$

So, the assertion is false.

Section B

21.

$6x^2 - 3 - 7x = 0$

$\Rightarrow 6x^2 - 7x - 3 = 0$

$\Rightarrow 6x^2 - 9x + 2x - 3 = 0$

$\Rightarrow 3x(2x-3) + (2x-3) = 0$

$\Rightarrow (3x+1)(2x-3) = 0$

$\Rightarrow 3x + 1 = 0$ or $2x - 3 = 0$

$\Rightarrow x = \frac{-1}{3}$ or $x = \frac{3}{2}$

So, the zeroes of $6x^2 - 3 - 7x$ are $\frac{-1}{3}$ and $\frac{3}{2}$.

Sum of zeroes $= \frac{-1}{3} + \frac{3}{2} = \frac{7}{6} = \frac{-(-7)}{6} = \frac{-(\text{Coefficient of } x)}{\text{Coefficient of } x^2}$

Product of zeroes $= \frac{-1}{3} \times \frac{3}{2} = \frac{-1}{2} = \frac{-3}{6} = \frac{\text{Constant term}}{\text{Coefficient of } x^2}$

22.

Given, DE || BC.
Then, by basic proportionality theorem,

$\frac{AD}{DB} = \frac{AE}{EC}$

$\frac{1.5}{3} = \frac{1}{EC}$

$EC = \frac{3 \times 1}{1.5} = 2 \text{ cm}$

OR

Given, DE || BC.
Then, by basic proportionality theorem,

$\frac{AD}{DB} = \frac{AE}{EC}$

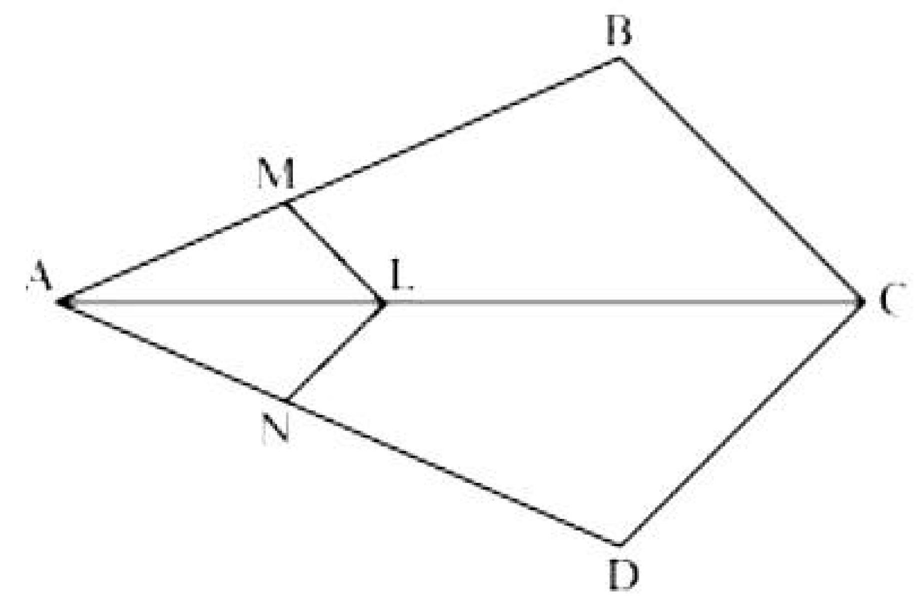
$\frac{AD}{7.2} = \frac{1.8}{5.4}$

$AD = \frac{1.8 \times 7.2}{5.4} = 2.4 \text{ cm}$

23.

In the given figure, LM || CB.

Then, by basic proportionality theorem,



$\frac{AM}{AB} = \frac{AL}{AC}$ (i)

Also LN || CD,

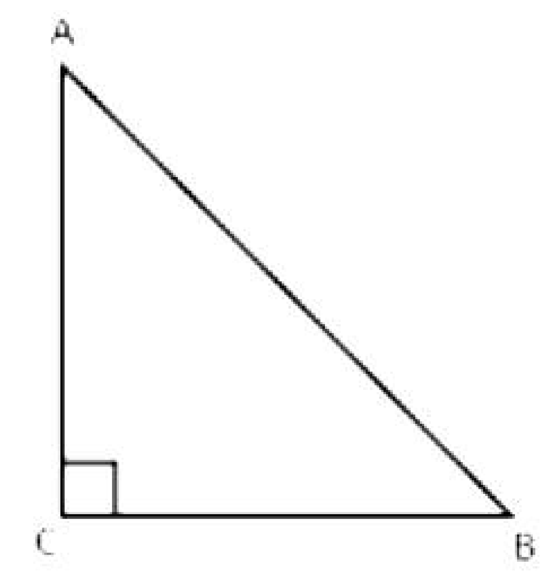
$\frac{AN}{AD} = \frac{AL}{AC}$ (ii)

From (i) and (ii)

$\frac{AM}{AB} = \frac{AN}{AD}$

24.

Given, cos A = cos B



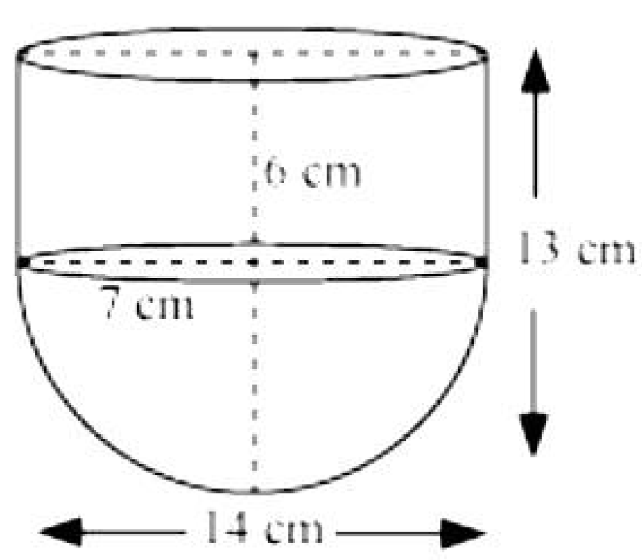
$\frac{AC}{AB} = \frac{BC}{AB}$

$\Rightarrow AC = BC$

So $\angle A = \angle B$ (Angles opposite to equal sides are equal in length)

25.

Radius (r) of cylindrical part and hemispherical part = 7 cm



Height of hemispherical part = radius = 7 cm.

Height of cylindrical part (h) = 13 – 7 = 6 cm

Inner surface area of the vessel

= CSA of cylindrical part + CSA of hemispherical part.

$= 2\pi rh + 2\pi r^2$

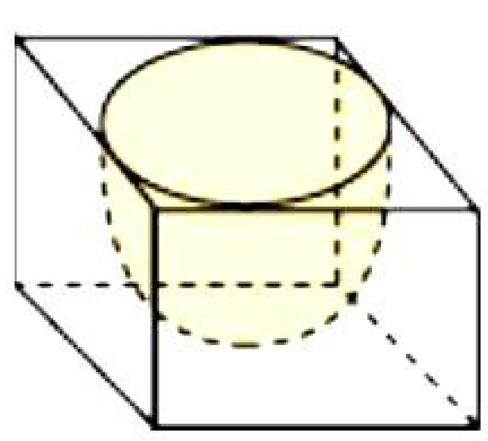
$= 2 \times \frac{22}{7} \times 7 \times 6 + 2 \times \frac{22}{7} \times 7 \times 7$

$= 44(6+7)$

$= 44 \times 13$

$= 572 \text{ cm}^2$

OR



Diameter of hemisphere = edge of cube = l

Radius of hemisphere = $\frac{l}{2}$

Total surface area of the remaining solid

= SA of cubical part + CSA of hemispherical part – area of base of hemispherical part

$= 6(\text{edge})^2 + 2\pi r^2 - \pi r^2$

$= 6(\text{edge})^2 + \pi r^2$

$$= 6l^2 + \pi \times \left(\frac{l}{2}\right)^2$$

$$= 6l^2 + \frac{\pi l^2}{4}$$

$$= l^2\left(6 + \frac{\pi}{4}\right)$$

$$= \frac{1}{4}l^2\left(\pi + 24\right) \text{ sq. units}$$

Section C

26. We have, $4^n = (2 \times 2)^n = 2^n \times 2^n$

The prime factors of 4^n are 2^n.

For any n, the number 2^n would end with a zero digit if the number is divisible by 5.

So, the prime factorisation should contain at least one prime factor 5.

But, $4^n = 2^n \times 2^n$, which contains the prime factor 2.

So, no other prime factorisation of 4^n exists due to the uniqueness of the Fundamental theorem.

Thus, any number of the form 4^n can never end with the digit 0.

27. Let the present age of Jacob be x years and the age of his son be y years.

According to the given information,

$(x+5) = 3(y+5)$

$x - 3y = 10 \qquad (1)$

$(x-5) = 7(y-5)$

$x - 7y = -30 \qquad (2)$

Subtracting (2) from (1),

$4y = 40 \Rightarrow y = 10$

Substituting this value in equation (1), we obtain

$x - 3(10) = 10 \Rightarrow x = 40$

Hence, the present age of Jacob is 40 years and that of his son is 10 years.

28. Let the units digit and tens digit of the number be x and y respectively.

$\therefore$ Number = 10y + x

Number obtained after reversing the digits = 10x + y

According to the question,

$x + y = 9$ ... (1)

And, $9(10y + x) = 2(10x + y)$

$\Rightarrow 90y + 9x = 20x + 2y$

$\Rightarrow 88y - 11x = 0$

$\Rightarrow -x + 8y = 0$ ... (2)

Adding equations (1) and (2),

$9y = 9$

$\Rightarrow y = 1$

Substituting the value of y in equation (1),

$x = 8$

Now, $10y + x = 10 \times 1 + 8 = 18$

Thus, the number is 18.

OR

Let the number of Rs. 50 notes and Rs. 100 notes be x and y respectively.

According to the question,

$x + y = 25$ ... (1)

$50x + 100y = 2000$ ... (2)

Multiplying equation (1) by 50, we get

$50x + 50y = 1250$ ... (3)

Subtracting equation (3) from equation (2), we have

$50y = 750$

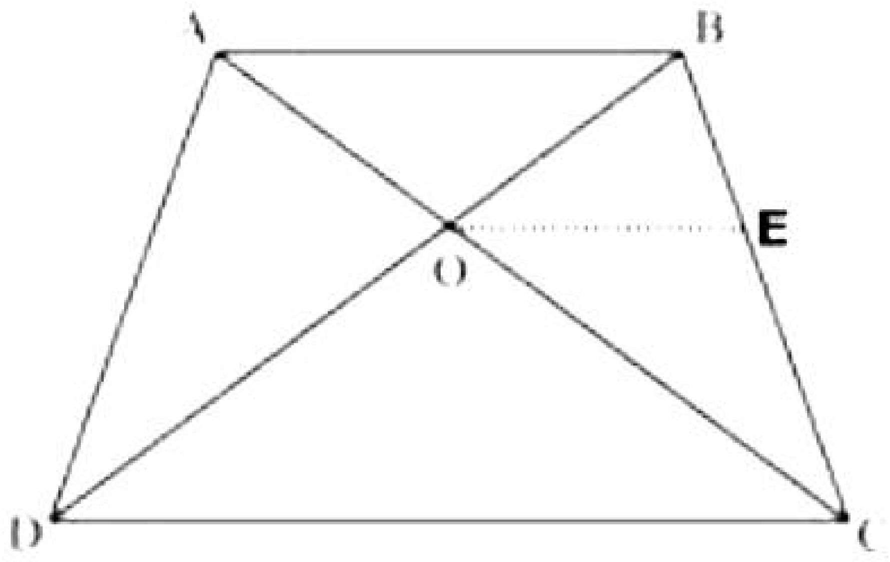
$\Rightarrow y = 15$

Substituting the value of y in equation (1),

$x = 10$

Hence, Meena received 10 notes of Rs. 50 and 15 notes of Rs. 100.

29.



Construction: Draw a line OE || AB.

In ΔABC, since OE || AB

$\therefore \frac{AO}{OC} = \frac{BE}{EC}$

But by the given relation, we have,

$$\frac{AO}{OC}=\frac{OB}{OD}$$

$$\therefore\ \frac{BE}{EC}=\frac{OB}{OD}$$

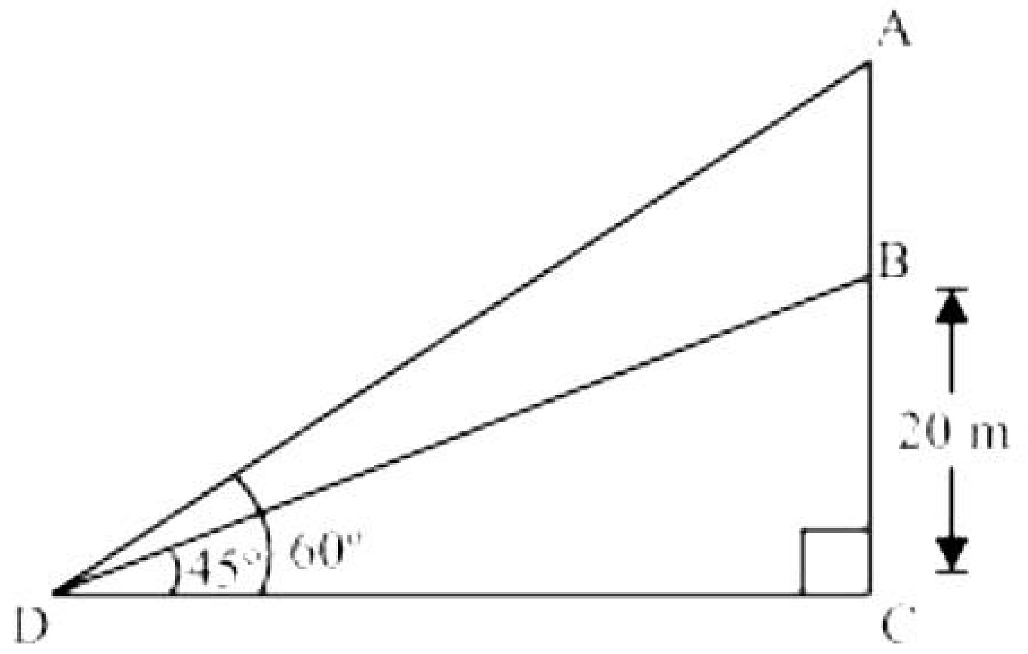
$\Rightarrow$ OE || DC

Therefore, AB || OE || DC

$\Rightarrow$ AB || CD

$\Rightarrow$ ABCD is a trapezium.

30.



Let BC be the building, AB be the transmission tower, and D be the point on the ground from where elevation angles are to be measured.

In ΔBCD,

$$\frac{BC}{CD}=\tan 45^\circ$$

$$\frac{20}{CD}=1$$

$$CD=20\text{ m} \qquad \text{... (i)}$$

In ΔACD,

$$\frac{AC}{CD}=\tan 60^\circ$$

$$\frac{AB+BC}{CD}=\sqrt{3}$$

$$\frac{AB+20}{20}=\sqrt{3} \qquad \text{[From(i)]}$$

$$AB=20\sqrt{3}-20$$

$$=20\left(\sqrt{3}-1\right)$$

Thus, the height of the tower is $20\left(\sqrt{3}-1\right)$ m.

OR

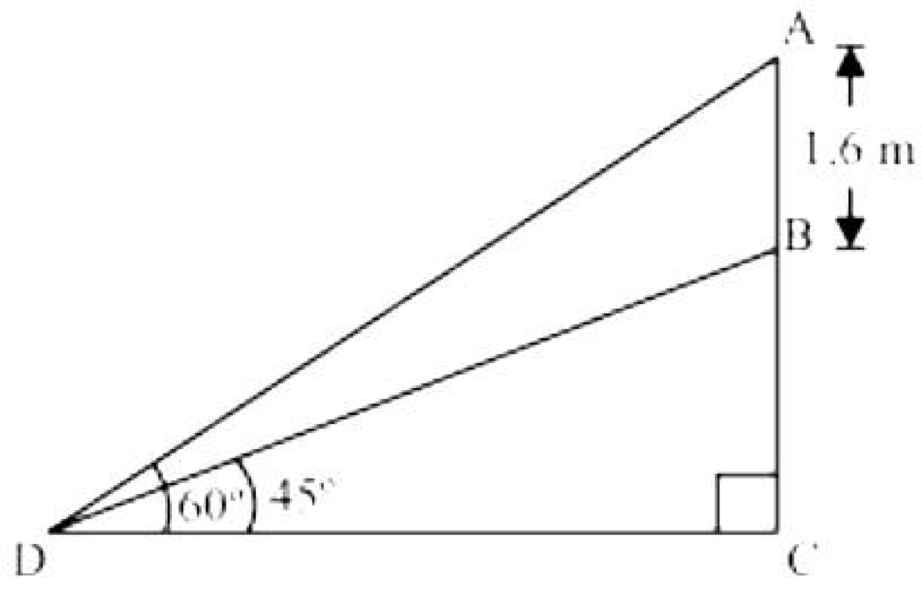
Let AB be the statue, BC be the pedestal and D be the point on ground from where elevation angles are to be measured.

In ΔBCD,

$$\frac{BC}{CD} = \tan 45° = 1$$

$$BC = CD \quad \text{... (i)}$$

In ΔACD,



$$\frac{AB + BC}{CD} = \tan 60°$$

$$\frac{AB + BC}{BC} = \sqrt{3} \quad \text{... [From (i)]}$$

$$1.6 + BC = BC\sqrt{3}$$

$$BC\left(\sqrt{3} - 1\right) = 1.6$$

$$BC = \frac{(1.6)\left(\sqrt{3}+1\right)}{\left(\sqrt{3}-1\right)\left(\sqrt{3}+1\right)}$$

$$= \frac{1.6\left(\sqrt{3}+1\right)}{2} = 0.8\left(\sqrt{3}+1\right)$$

Thus, the height of pedestal is $0.8\left(\sqrt{3}+1\right)$ m.

31. Class size (h) of this data = 20

Now taking 150 as assured mean (a), we may calculate d_i, u_i and f_iu_i as follows:

Daily wages (in Rs)	Number of workers (f_i)	x_i	$d_i = x_i - 150$	$u_i = \frac{x_i - 150}{h}$	f_iu_i
100 – 120	12	110	–40	–2	–24
120 – 140	14	130	–20	–1	–14
140 – 160	8	150	0	0	0
160 – 180	6	170	20	1	6
180 – 200	10	190	40	2	20
Total	50				–12

From the table, $\sum f_i = 50$ and $\sum f_iu_i = -12$

$$\text{Mean}\left(\bar{x}\right) = a + \left(\frac{\sum f_iu_i}{\sum f_i}\right)h$$

$$= 150 + \left(\frac{-12}{50}\right)20$$

$$= 150 - \frac{24}{5}$$

$$= 145.2$$

So, the mean daily wages of the workers in a factory is Rs.145.20.

Section D

32.

Given, $a_3 = 4$ and $a_9 = -8$

We know that, $a_n = a + (n - 1) d$

$a_3 = a + (3 - 1)d$

$4 = a + 2d$ (I)

$a_9 = a + (9 - 1)d$

$-8 = a + 8d$ (II)

On subtracting equation (I) from (II), we obtain

$-12 = 6d$

$d = -2$

From equation (I), we obtain

$4 = a + 2(-2)$

$4 = a - 4$

$a = 8$

Let the n^{th} term of this A.P. be zero.

$a_n = a + (n - 1)d$

$0 = 8 + (n - 1)(-2)$

$0 = 8 - 2n + 2$

$2n = 10$

$n = 5$

Hence, the 5^{th} term of this A.P. is 0.

OR

Let the first term of these A.P.s be a_1 and a_2 respectively and the common difference of these A.P.s be d.

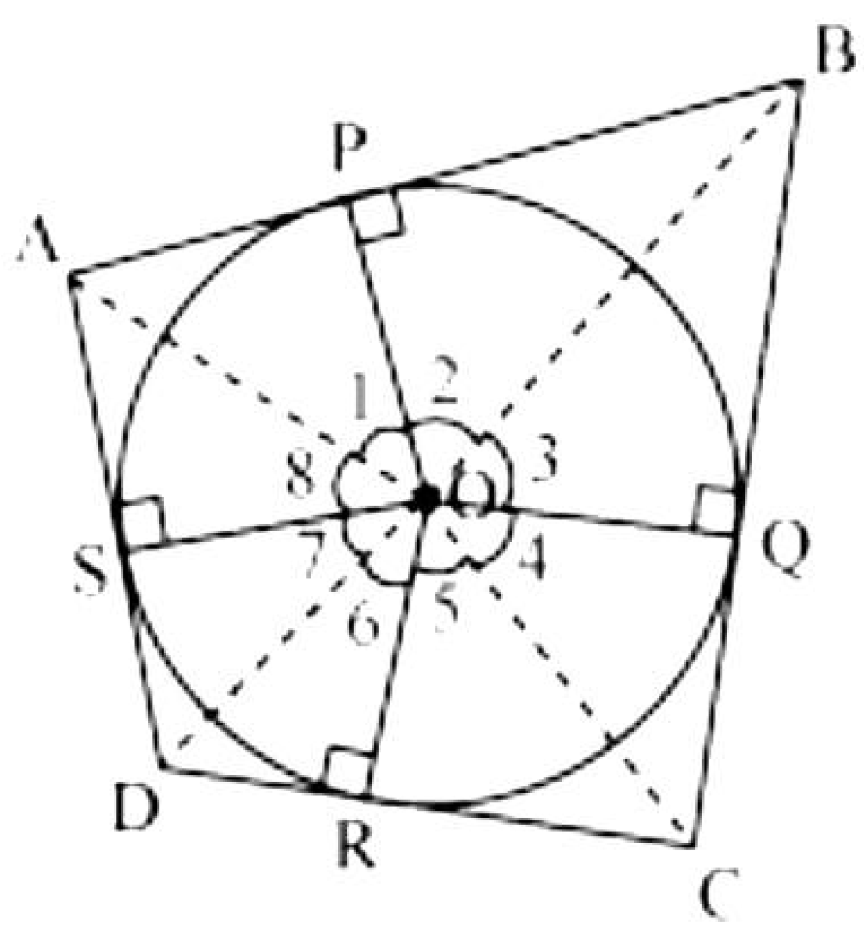
For first A.P.,

$a_{100} = a_1 + (100 - 1)d$

$= a_1 + 99d$

$a_{1000} = a_1 + (1000 - 1)d$
$a_{1000} = a_1 + 999d$
For second A.P.,
$a_{100} = a_2 + (100 - 1)d$
$= a_2 + 99d$
$a_{1000} = a_2 + (1000 - 1)d = a_2 + 999d$
Given that, difference between 100^{th} terms of these A.P.s = 100
Therefore, $(a_1 + 99d) - (a_2 + 99d) = 100$
$a_1 - a_2 = 100$ (1)
Difference between 1000^{th} terms of these Aps
$= (a_1 + 999d) - (a_2 + 999d) = a_1 - a_2$
From equation (1),
This difference, $a_1 - a_2 = 100$
Hence, the difference between 1000^{th} terms of these A.P.s will be 100.

33.



Let ABCD be a quadrilateral circumscribing a circle centered at O such that it touches the circle at points P, Q, R, S.
Join the vertices of the ABCD to the center of the circle.
Consider ΔOAP and ΔOAS,
AP = AS (tangents from the same point)
OP = OS (radii of the same circle)
OA = OA (common)
So, ΔOAP ≅ ΔOAS (SSS congruence rule)
$\therefore \angle POA = \angle SOA$
$\angle 1 = \angle 8$
Similarly,
$\angle 2 = \angle 3$
$\angle 4 = \angle 5$
$\angle 6 = \angle 7$
$\angle 1 + \angle 2 + \angle 3 + \angle 4 + \angle 5 + \angle 6 + \angle 7 + \angle 8 = 360°$
$(\angle 1 + \angle 8) + (\angle 2 + \angle 3) + (\angle 4 + \angle 5) + (\angle 6 + \angle 7) = 360°$

$2\angle 1 + 2\angle 2 + 2\angle 5 + 2\angle 6 = 360°$

$2(\angle 1 + \angle 2) + 2(\angle 5 + \angle 6) = 360°$

$(\angle 1 + \angle 2) + (\angle 5 + \angle 6) \quad = 180°$

$\angle AOB + \angle COD = 180°$

Similarly, $\angle BOC + \angle DOA = 180°$

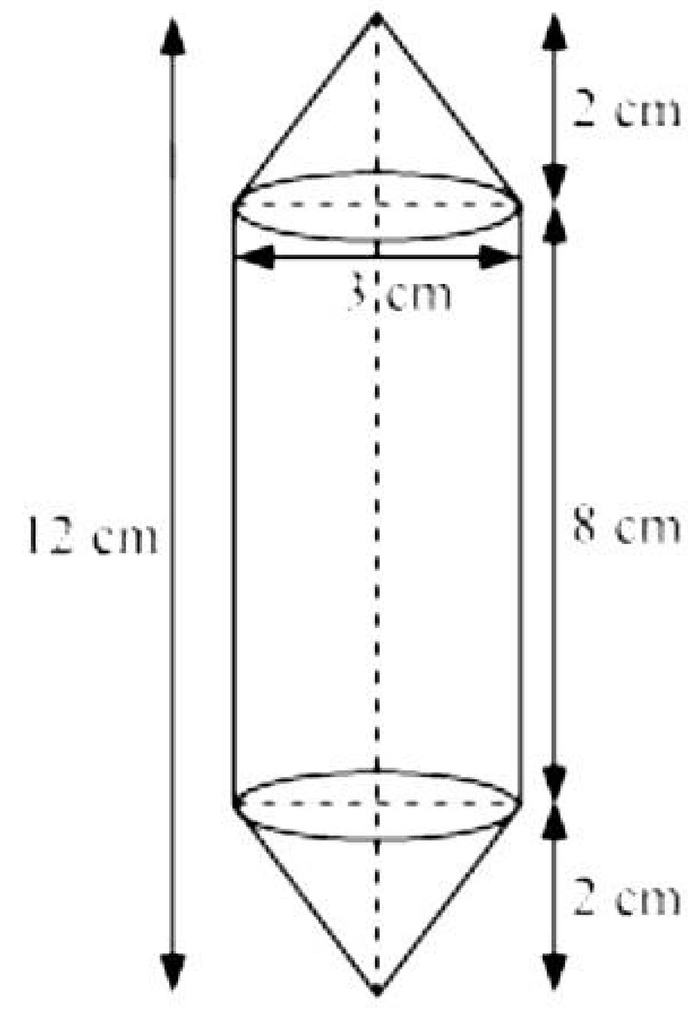
Hence, the opposite sides of a quadrilateral circumscribing a circle subtend supplementary angles at the centre of the circle.

34.

From the figure Height (h_1) of each conical part = 2 cm

Height (h_2) of cylindrical part = 12 – 2 × height of conical part

= 12 – 2 × 2 = 8 cm



Radius (r) of cylindrical part = radius of conical part = $\frac{3}{2}$ cm

Volume of air present in the model

= volume of cylinder + 2 × volume of cone

$= \pi r^2 h_2 + 2 \times \frac{1}{3}\pi r^2 h_1$

$= \pi\left(\frac{3}{2}\right)^2 (8) + 2 \times \frac{1}{3}\pi\left(\frac{3}{2}\right)^2 (2)$

$= \pi \times \frac{9}{4} \times 8 + \frac{2}{3}\pi \times \frac{9}{4} \times 2$

$= 18\pi + 3\pi$

$= 21\pi$

$= 21 \times \frac{22}{7}$

$= 66 \text{ cm}^3$

OR

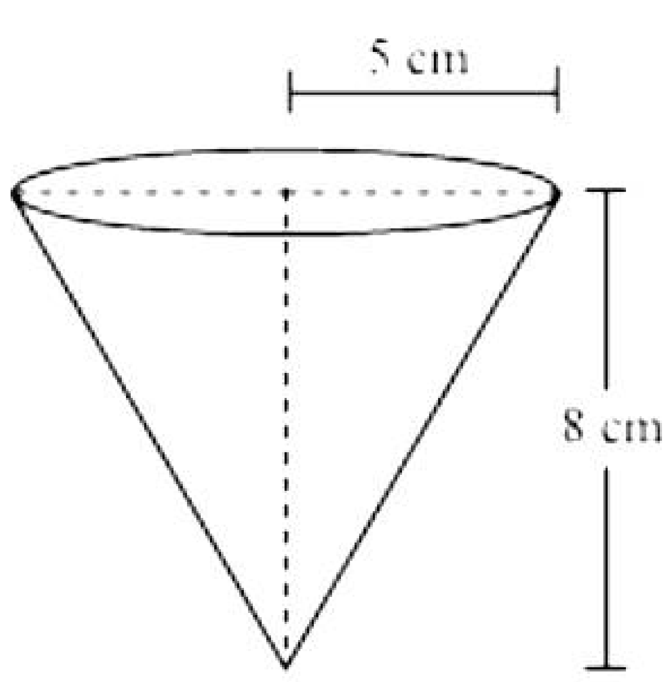
Height (h) of conical vessel = 8 cm

Radius (r_1) of conical vessel = 5 cm

Radius (r_2) of lead shots = 0.5 cm

Let 'n' number of lead shots be dropped in the vessel.

Volume of water spilled = Volume of dropped lead shots



$\frac{1}{4} \times \text{volume of cone} = n \times \frac{4}{3} \pi r_2^3$

$\frac{1}{4} \times \frac{1}{3} \pi r_1^2 h = n \times \frac{4}{3} \pi r_2^3$

$r_1^2 h = n \times 16 r_2^3$

$5^2 \times 8 = n \times 16 \times (0.5)^3$

$n = \frac{25 \times 8}{16 \times \left(\frac{1}{2}\right)^3} = 100$

Hence, the number of lead shots dropped in the vessel is 100.

35. We can find cumulative frequencies with their respective class intervals as below:

Number of letters	Frequency (f_i)	Cumulative frequency
1 – 4	6	6
4 – 7	30	30 + 6 = 36
7 – 10	40	36 + 40 = 76
10 – 13	16	76 + 16 = 92
13 – 16	4	92 + 4 = 96
16 – 19	4	96 + 4 = 100
Total (n)	100	

Now we may observe that cumulative frequency just greater than $\frac{n}{2}\left(\text{i.e. } \frac{100}{2} = 50\right)$ is 76 belonging to class interval 7 – 10.

Median class = 7 – 10

Lower limit (l) of median class = 7

Cumulative frequency (cf) of class preceding median class = 36

Frequency (f) of median class = 40

Class size (h) = 3

Median $= l + \left(\frac{\frac{n}{2} - cf}{f}\right) \times h$

$= 7 + \left(\frac{50 - 36}{40}\right) \times 3$

$= 7 + \frac{14 \times 3}{40}$

= 8.05

Now we can find class marks of given class intervals by using relation.

$$\text{Class mark} = \frac{\text{Upper class limit} + \text{Lower class limit}}{2}$$

Taking 11.5 as assumed mean (a) we can find d_i, u_i and f_iu_i according to step deviation method as below.

Number of letters	Number of surnames	x_i	$x_i - a$	$u_i = \frac{x_i - a}{3}$	f_iu_i
1 – 4	6	2.5	−9	−3	−18
4 – 7	30	5.5	−6	−2	−60
7 – 10	40	8.5	−3	−1	−40
10 – 13	16	11.5	0	0	0
13 – 16	4	14.5	3	1	4
16 – 19	4	17.5	6	2	8
Total	100				−106

$\sum f_iu_i = -106$ and $\sum f_i = 100$

Mean $\bar{x} = a + \left(\frac{\sum f_iu_i}{\sum f_i}\right)h$

$= 11.5 + \left(\frac{-106}{100}\right) \times 3$

$= 11.5 - 3.18$

$= 8.32$

Now,

3(median) = mode + 2(mean)

3(8.05) = mode + 2(8.32)

24.15 – 16.64 = mode

7.51 = mode

So, median number and mean number of letters in surnames is 8.05 and 8.32 respectively while modal size of surnames is 7.51.

Section E

36.

i. Monthly payments made by Aarushi are:
Rs. 1000, Rs. 1100, Rs. 1200, Rs. 1300, ...
Terms of the above sequence forms an A.P.
Here, a = 1000 and d = 100
Therefore, amount paid in 30^{th} months = a_{30}
$\Rightarrow a_{30} = 1000 + 29(100) = 1000 + 2900 =$ Rs. 3900

ii. Let Aarushi paid Rs. 4900 as instalment in n^{th} month.
Now, $a_n = a + (n - 1)d$
$\Rightarrow 4900 = 1000 + (n - 1)100$
$\Rightarrow (n - 1)100 = 3900$
$\Rightarrow n = 40$
Thus, Aarushi paid Rs. 4900 in 40^{th} month.

iii. Here, $a_{19} = 1000 + 18(100) = 2800$
Also, $a_{28} = 1000 + 27(100) = 3700$
Ratio of 19^{th} term to 28^{th} terms = 2800 : 3700
= 28 : 37

OR

Here, a = 1000 and d = 100

$$S_{30} = \frac{30}{2}\left[2(1000)+(30-1)100\right]$$
$$= 15\left[2000+2900\right]$$
$$= 15\times 4900$$
$$= \text{Rs.}73{,}500$$

37.

i.

$A(4,1)$ and $B(7,1)$

$$d(AB) = \sqrt{(7-4)^2+(1-1)^2} = 3 \text{ km}$$

ii.

$C(7,5)$ and $B(7,1)$

$$d(BC) = \sqrt{(7-7)^2+(5-1)^2} = 4 \text{ km}$$

iii.

$C(7,5)$ and $A(4,1)$

$d(CA)=\sqrt{(7-4)^2+(5-1)^2}=5\text{ km}$

OR

Longest route (via Bipin's home) $\rightarrow$ A-B-C = 7 km

Shortest route $\rightarrow$ A-C

$C(7,5)$ and $A(4,1)$

$d(CA)=\sqrt{(7-4)^2+(5-1)^2}=5\text{ km}$

Difference = 2 km.

38.

i.

In ΔABH,

$\tan 45^\circ=\frac{AB}{BH}$

$\therefore 1=\frac{10}{BH}$

$\therefore BH=10\text{ m}=\text{Distance between Harsh and building AB}$

ii.

In ΔCDH,

$\tan 60^\circ=\frac{CD}{DH}$

$\therefore \sqrt{3}=\frac{15}{DH}$

$\therefore DH=5\sqrt{3}\text{ m}=\text{Distance between Harsh and building CD}$

iii.

In ΔABH,

$\sin 45^\circ=\frac{AB}{AH}$

$\therefore \frac{1}{\sqrt{2}}=\frac{10}{AH}$

$\therefore AH=10\sqrt{2}\text{ m}=\text{Length of a rope joining points A and H}$

OR

In ΔCDH,

$\sin 60^\circ=\frac{CD}{HC}$

$\therefore \frac{\sqrt{3}}{2}=\frac{15}{HC}$

$\therefore HC=10\sqrt{3}\text{ m}=\text{Length of a rope joining points C and H}$